

# Some Properties Associated with Clifford-Fourier Transform

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# Some Properties Associated with Clifford-Fourier Transform

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**Abstract.** Several useful properties of the Clifford-Fourier transform have been studied recently and many are being investigated at present. In this paper, we explore more properties of the Clifford-Fourier transform. We find that the properties are extensions of corresponding properties of the classical Fourier transform.

## 1. Introduction

In recent years, the work related the Clifford-Fourier transform (CFT) has grown rapidly. At first, the CFT was introduced by Brackx et al. [1] who proposed to extend the classical Fourier transformation [2] to Clifford analysis  $Cl_{0,n}$ . Several fundamental properties of this generalized transformation were studied. Further, the application of the CFT to vector fields and the behaviour of vector-valued filters has been investigated by Ebling and Scheuermann [3]. As far, there are various types of the Clifford-Fourier transforms have been proposed by the researcher. In [4, 5, 6, 7] the authors developed the CFT of the reference [3] to higher dimensions and obtained fundamental properties like convolution, correlation and uncertainty principle. The CFT of this approach has used the authors [8, 9, 10] to constructed windowed Fourier and wavelet transformations in the setting of Clifford algebra. The different approach of the CFT has been proposed the authors [11]. Some results related to this CFT has been published in [12, 13, 14]. Again in [15] Hitzer has proposed the new type of the CFT which it can be considered as a general form of the double-sided quaternion Fourier transformation [16, 17, 18]. Some important result related to new transformation such as convolution and correlation were investigated in detail.

The main purpose of this work is to investigate the several results of the CFT which do not have been published in the literature. To achieve this we first introduce the definition of the CFT. Some results related to the properties of the CFT kernel are presented. We then derive the duality property associated with the CFT. We finally show that under certain condition the Clifford function related to the CFT is continuous and bounded.

The remainder of the present paper is organized as follows. Section 2, some preliminary results related to Clifford algebra are discussed. The result will be used in the sequel. In Section 3 we introduce the definition the CFT and obtain some results related to the CFT

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kernel. Section 4 derive in detail the Clifford function related to the CFT is continuous and bounded.

## 2. Clifford Algebra <sup>6</sup>

Let  $\{e_1, e_2, e_3, \dots, e_n\}$  be an orthonormal basis of the real  $n$ -dimensional space  $\mathbb{R}^{(r,t)}$  with  $r + t = n$ . The real Clifford algebra (see [19, 20]) over  $\mathbb{R}^{(r,t)}$  is denoted by  $Cl_{(r,t)}$  such that

$$\{1, e_1, e_2, \dots, e_n, e_{12}, e_{23}, \dots, e_1 e_2 \dots e_n\}, \quad (1)$$

<sup>10</sup> where  $I$  is a unit oriented pseudoscalar. The product of the above basis vectors fulfills the following rules:

$$\begin{aligned} e_k e_l &= -e_l e_k && \text{for } k \neq l, \quad 1 \leq k, l \leq n, \\ e_k^2 &= 1 && \text{for } 1 \leq k \leq r \\ e_k^2 &= -1 && \text{for } r + 1 \leq k \leq n. \end{aligned} \quad (2)$$

The elements of Clifford algebra are called multivectors. It means that every  $g \in Cl_{(r,t)}$  may be expressed as

$$g = \sum_C g_C e_C, \quad (3)$$

where  $g_C \in \mathbb{R}$ ,  $e_C = e_{\alpha_1 \alpha_2 \dots \alpha_k} = e_{\alpha_1} e_{\alpha_2} \dots e_{\alpha_k}$ , and  $1 \leq \alpha_1 < \alpha_2 < \dots < \alpha_k \leq n$  with  $\alpha_j \in \{1, 2, \dots, n\}$ . For simplicity, we write  $\langle g \rangle_l = \sum_{|C|=l} g_C e_C$  to denote  $l$ -vector part of  $g$  ( $l = 0, 1, 2, \dots, n$ ), then

$$g = \sum_{l=0}^{l=n} \langle g \rangle_l = \langle g \rangle + \langle g \rangle_1 + \langle g \rangle_2 + \dots + \langle g \rangle_n, \quad (4)$$

where  $\langle \dots \rangle_0 = \langle \dots \rangle$ .

The reverse  $\bar{g}$  of a multivector  $g$  is given by

$$\bar{g} = \sum_{l=0}^{l=n} (-1)^{l(l-1)/2} \langle g \rangle_l, \quad (5)$$

which satisfies  $\overline{\bar{h}} = h$  for every  $g, h \in Cl_{(r,t)}$ .

The scalar product of multivectors  $f, \bar{g}$  is defined as the scalar part of the geometric product  $f \bar{g}$  of multivectors

$$\langle g \bar{h} \rangle = g \star \bar{h} = \sum_C g_C h_C. \quad (6)$$

Notice that if  $g = h$  in (6), then we get the modulus  $|g|$  of a multivector  $g \in Cl_{(r,t)}$  given by

$$|g|^2 = g \star \bar{g} = \sum_C g_C^2. \quad (7)$$

It is not difficult to see that for  $g, h \in Cl_{(r,t)}$  one can obtain

$$|gh|^2 \leq 2^n |g|^2 |h|^2. \quad (8)$$

**Definition 2.1.** A multivector  $g \in Cl_{(r,t)}$  is called vectorial if it takes the form

$$g = g_0 + e_1 g_1 + e_2 g_2 + \dots + e_n g_n. \quad (9)$$

We define the inner product for multivector functions  $g, h : \mathbb{R}^{(r,t)} \rightarrow Cl_{(r,t)}$  as follows:

$$\begin{aligned} (g, h) &= \int_{\mathbb{R}^{(r,t)}} g(\mathbf{x}) \overline{h(\mathbf{x})} d^n \mathbf{x} \\ &= \sum_{C,D} e_C \overline{e_D} \int_{\mathbb{R}^{(r,t)}} g_C(\mathbf{x}) h_D(\mathbf{x}) d^n \mathbf{x}, \quad d^n \mathbf{x} = dx_1 dx_2 \cdots dx_n. \end{aligned} \quad (10)$$

Thus for  $g = h$  we get

$$\|g\|^2 \stackrel{(6)}{=} \int_{\mathbb{R}^{(r,t)}} \sum_C g_C^2(\mathbf{x}) d^n \mathbf{x}. \quad (11)$$

### 3. Clifford-Fourier Transform (CFT)

In what follows, we provide the definition of the Clifford-Fourier transform (CFT) and its inverse. We also demonstrate an important property of the CFT kernel.

**Definition 3.1.** Let  $I \in Cl_{(r,t)}$  be a square root of  $-1$  such that  $I^2 = -1$ . The CFT of  $g \in L^1(\mathbb{R}^{(r,t)}; Cl_{(r,t)})$  is given by

$$\mathcal{F}_{Cl}\{g\}(\mathbf{u}) = \hat{g}(\mathbf{u}) = \frac{1}{(2\pi)^{\frac{n}{2}}} \int_{\mathbb{R}^{(r,t)}} g(\mathbf{x}) e^{-Iv(\mathbf{u},\mathbf{x})} d^n \mathbf{x}, \quad (12)$$

with  $\mathbf{x}, \mathbf{u} \in \mathbb{R}^{(r,t)}$  and  $v : \mathbb{R}^{(r,t)} \times \mathbb{R}^{(r,t)} \rightarrow \mathbb{R}$ .

In the remainder of the paper, we always assume that

$$v(\mathbf{u}, \mathbf{x}) = u_1 x_1 + u_2 x_2 + \cdots + u_n x_n. \quad (13)$$

**Lemma 3.1.** [13] For any  $g \in Cl_{(r,t)}$  and  $I \in Cl_{(r,t)}$ , one can get

$$|e^{-Iv(\mathbf{u},\mathbf{x})}| \leq (1 + |I|^2)^{\frac{1}{2}}, \quad (14)$$

and

$$|g e^{-Iv(\mathbf{u},\mathbf{x})}| \leq 2^n |g| (1 + |I|^2)^{\frac{1}{2}}. \quad (15)$$

**Definition 3.2.** For arbitrary  $g \in L^1(\mathbb{R}^{(r,t)}; Cl_{(r,t)})$  we define the inverse of the CFT as

$$\mathcal{F}_{Cl}^{-1}\{g\}(\mathbf{x}) = \frac{1}{(2\pi)^{\frac{n}{2}}} \int_{\mathbb{R}^{(r,t)}} g(\mathbf{u}) e^{Iv(\mathbf{u},\mathbf{x})} d^n \mathbf{u}. \quad (16)$$

**Theorem 3.2.** For  $f \in L^1(\mathbb{R}^{(r,t)}; Cl_{(r,t)})$ , it holds

$$\mathcal{F}_{Cl}^{-1}[\mathcal{F}_{Cl}\{g\}](\mathbf{u}) = g(\mathbf{u}). \quad (17)$$

### 4. New results for CFT

We first investigate a number of useful properties of the CFT, which can be regarded as extensions of the results of the classical Fourier transformation [2].

**Theorem 4.1.** For  $h, g \in L^2(\mathbb{R}^{(r,t)}; Cl_{(r,t)})$ , it holds that

$$(\mathcal{F}_{Cl}\{h\}, g) = (h, \mathcal{F}_{Cl}^{-1}\{g\}). \quad (18)$$

*Proof.* In fact, one has

$$\begin{aligned}
 (\mathcal{F}_{Cl}\{h\}, g) &= \int_{\mathbb{R}^{(r,t)}} \mathcal{F}_{Cl}\{h\}(\mathbf{u}) \overline{g(\mathbf{u})} d\mathbf{u} \\
 &= \int_{\mathbb{R}^{(r,t)}} \left[ \int_{\mathbb{R}^{(r,t)}} h(\mathbf{x}) e^{-Iv(\mathbf{u},\mathbf{x})} d^n \mathbf{x} \right] \overline{g(\mathbf{u})} d^n \mathbf{u} \\
 &= \int_{\mathbb{R}^{(r,t)}} \int_{\mathbb{R}^{(r,t)}} h(\mathbf{x}) e^{-Iv(\mathbf{u},\mathbf{x})} \overline{g(\mathbf{u})} d^n \mathbf{x} d^n \mathbf{u} \\
 &= \int_{\mathbb{R}^{(r,t)}} \int_{\mathbb{R}^{(r,t)}} h(\mathbf{x}) g(\mathbf{u}) e^{Iv(\mathbf{u},\mathbf{x})} d^n \mathbf{u} d^n \mathbf{x} \\
 &= \int_{\mathbb{R}^{(r,t)}} h(\mathbf{x}) \overline{\mathcal{F}_{Cl}^{-1}\{g\}(\mathbf{x})} d^n \mathbf{x} \\
 &= (h, \mathcal{F}_{Cl}^{-1}\{g\}), \tag{19}
 \end{aligned}$$

which this is the desired result.  $\square$

**Definition 4.1.** Let  $\mathcal{F}_{Cl}$  be the Clifford-Fourier transformation. The adjoint of  $\mathcal{F}_{Cl}$  is denoted by  $\mathcal{F}_{Cl}^*$  and is defined by

$$(\mathcal{F}_{Cl}\{h\}, g) = (h, \mathcal{F}_{Cl}^*\{g\}). \tag{20}$$

The following result demonstrates the relationship between adjoint of CFT and its inverse.

**Theorem 4.2.** Let  $h, g \in L^2(\mathbb{R}^{(r,t)}; Cl_{(r,t)})$ . The adjoint of the CFT is its inversion formula, that is,

$$(\mathcal{F}_{Cl}\{h\}, g) = (h, \mathcal{F}_{Cl}^{-1}\{g\}). \tag{21}$$

*Proof.* By combining (18) and (20) we can finish the proof.  $\square$

**Theorem 4.3** (Parseval's formula for  $\mathcal{F}_{Cl}^*$ ). If  $h, g \in L^2(\mathbb{R}^{(r,t)}; Cl_{(r,t)})$ , then the following is satisfied

$$(\mathcal{F}_{Cl}^*\{h\}, \mathcal{F}_{Cl}^*\{g\}) = (h, g). \tag{22}$$

*Proof.* By (21) we obtain

$$\begin{aligned}
 (\mathcal{F}_{Cl}^*\{h\}, \mathcal{F}_{Cl}^*\{g\}) &= (\mathcal{F}_{Cl}^{-1}\{h\}, \mathcal{F}_{Cl}^{-1}\{g\}) \\
 &= \int_{\mathbb{R}^{(r,t)}} \mathcal{F}_{Cl}^{-1}\{h\}(\mathbf{x}) \overline{\mathcal{F}_{Cl}^{-1}\{g\}(\mathbf{x})} d^n \mathbf{x} \\
 &= \int_{\mathbb{R}^{(r,t)}} \left( \int_{\mathbb{R}^{(p,q)}} h(\mathbf{u}) e^{Iv(\mathbf{u},\mathbf{x})} d^n \mathbf{u} \right) \overline{\mathcal{F}_{Cl}^{-1}\{g\}(\mathbf{x})} d^n \mathbf{x} \\
 &= \int_{\mathbb{R}^{(r,t)}} \left( \int_{\mathbb{R}^{(p,q)}} \mathcal{F}_{Cl}\{h\}(\mathbf{u}) e^{Iv(\mathbf{u},\mathbf{x})} d^n \mathbf{u} \right) \overline{\mathcal{F}_{Cl}^{-1}\{g\}(\mathbf{x})} d^n \mathbf{x} \\
 &= \int_{\mathbb{R}^{(r,t)}} \mathcal{F}_{Cl}\{h\}(\mathbf{u}) \left( \int_{\mathbb{R}^{(p,q)}} \overline{g(\mathbf{x})} e^{Iv(\mathbf{u},\mathbf{x})} d^n \mathbf{x} \right) d^n \mathbf{u} \\
 &= \int_{\mathbb{R}^{(r,t)}} \mathcal{F}_{Cl}\{h\}(\mathbf{u}) \overline{\mathcal{F}_{Cl}\{g\}(\mathbf{u})} d^n \mathbf{u} \\
 &= (\mathcal{F}_{Cl}\{h\}, \mathcal{F}_{Cl}\{g\}) \\
 &= (h, g), \tag{23}
 \end{aligned}$$

which gives the required result.  $\square$

**Theorem 4.4.** Given  $h \in L^2(\mathbb{R}^{(r,t)}; Cl_{(r,t)})$  and  $g = \mathcal{F}_{Cl}\{h\}$ . If we assume that

$$\langle h, \mathcal{F}_{Cl}\{g\} \rangle = \langle \mathcal{F}_{Cl}\{h\}, g \rangle. \quad (24)$$

Then we have

$$h = \mathcal{F}_{Cl}\{g\}. \quad (25)$$

*Proof.* From the hypothesis of the theorem and the Parseval's theorem for the CFT, we see that

$$\langle h, \mathcal{F}_{Cl}\{g\} \rangle = \langle \mathcal{F}_{Cl}\{h\}, g \rangle = \langle \mathcal{F}_{Cl}\{h\}, \mathcal{F}_{Cl}\{h\} \rangle = \|\mathcal{F}_{Cl}\{h\}\|_2^2 = \|h\|^2. \quad (26)$$

Applying Parseval's formula results in

$$\|\mathcal{F}_{Cl}\{g\}\|^2 = \|g\|^2 = \|\mathcal{F}_{Cl}\{h\}\|^2 = \|h\|^2. \quad (27)$$

Consequently we get

$$\begin{aligned} \|h - \mathcal{F}_{Cl}\{g\}\|^2 &= \langle h - \mathcal{F}_{Cl}\{g\}, h - \mathcal{F}_{Cl}\{g\} \rangle \\ &= \|h\|^2 - \langle h, \mathcal{F}_{Cl}\{g\} \rangle - \langle \mathcal{F}_{Cl}\{g\}, h \rangle + \|\mathcal{F}_{Cl}\{g\}\|^2 \\ &= \|h\|^2 - \|\mathcal{F}_{Cl}\{h\}\|^2 - \|h\|^2 + \|\mathcal{F}_{Cl}\{g\}\|^2 \\ &= 0. \end{aligned} \quad (28)$$

This proves the theorem.  $\square$

**Theorem 4.5** (CFT duality). Let  $\mathcal{F}_{Cl}\{g\}$  be a CFT of Clifford function  $g$ . Then we get

$$\mathcal{F}_{Cl}\{\hat{g}(\mathbf{u})\} = g(-\mathbf{u}). \quad (29)$$

*Proof.* It directly follows from (12) that

$$\begin{aligned} \mathcal{F}_{Cl}\{\hat{g}(\mathbf{u})\} &= \frac{1}{(2\pi)^{\frac{n}{2}}} \int_{\mathbb{R}^{(r,t)}} \mathcal{F}_{Cl}\{g\}(\mathbf{u}) e^{-Iv(\mathbf{u}, \mathbf{x})} d^n \mathbf{x} \\ &= \frac{1}{(2\pi)^{\frac{n}{2}}} \int_{\mathbb{R}^{(r,t)}} \mathcal{F}_{Cl}\{g\}(\mathbf{x}) e^{-Iv(\mathbf{u}, \mathbf{x})} d^n \mathbf{u} \\ &= g(-\mathbf{u}). \end{aligned} \quad (30)$$

Then one has

$$\mathcal{F}_{Cl}\{\hat{g}(-\mathbf{u})\} = g(\mathbf{u}). \quad (31)$$

Now observe that

$$\begin{aligned} \mathcal{F}_{Cl}\{\mathcal{F}_{Cl}\{\hat{g}(\mathbf{u})\}\} &= \frac{1}{(2\pi)^{\frac{n}{2}}} \int_{\mathbb{R}^{(r,t)}} g(-\mathbf{u}) e^{-Iv(\mathbf{u}, \mathbf{x})} d^n \mathbf{u} \\ &= \frac{1}{(2\pi)^{\frac{n}{2}}} \int_{\mathbb{R}^{(r,t)}} g(\mathbf{z}) e^{Iv(\mathbf{z}, \mathbf{x})} d^n \mathbf{z} \\ &= \frac{1}{(2\pi)^{\frac{n}{2}}} \int_{\mathbb{R}^{(r,t)}} g(\mathbf{t}) e^{Iv(\mathbf{t}, \mathbf{u})} d^n \mathbf{t} \\ &= \mathcal{F}_{Cl}\{g\}(-\mathbf{u}). \end{aligned} \quad (32)$$

This is the desired result.  $\square$

**Theorem 4.6** (Continuity). *If  $g \in L^1(\mathbb{R}^{(r,t)}; Cl_{(r,t)})$  then  $\mathcal{F}_{Cl}\{g\}$  is continuous and bounded on  $\mathbb{R}^{(r,t)}$ .*

*Proof.* For every  $\xi, u \in \mathbb{R}^{(r,t)}$  we obtain

$$\begin{aligned} |\mathcal{F}_{Cl}\{g\}(u)| &\leq \frac{1}{(2\pi)^{\frac{n}{2}}} \int_{\mathbb{R}^{(r,t)}} |g(x)e^{-Iv(u,x)}| d^n x \\ &= \frac{1}{(2\pi)^{\frac{n}{2}}} \int_{\mathbb{R}^{(r,t)}} |g(x)e^{-Iv(u,x)}| d^n x \\ &\leq \left(\frac{2}{\pi}\right)^{\frac{n}{2}} (1 + |I|^{\frac{1}{2}}) \int_{\mathbb{R}^{(r,t)}} |g(x)| d^n x. \end{aligned} \tag{33}$$

This gives  $\mathcal{F}_{Cl}\{g\}$  is bounded. Next, using the CFT definition (12), we have

$$\begin{aligned} &|\mathcal{F}_{Cl}\{g\}(u + \xi) - \mathcal{F}_{Cl}\{g\}(u)| \\ &= \left| \frac{1}{(2\pi)^{\frac{n}{2}}} \int_{\mathbb{R}^{(r,t)}} g(x)e^{-Iv(u+\xi,x)} d^n x - \frac{1}{(2\pi)^{\frac{n}{2}}} \int_{\mathbb{R}^{(r,t)}} g(x)e^{-Iv(u,x)} d^n x \right| \\ &= \left| \frac{1}{(2\pi)^{\frac{n}{2}}} \int_{\mathbb{R}^{(r,t)}} g(x)e^{-Iv(u,x)} (e^{-Iv(\xi,x)} - 1) d^n x \right| \\ &\leq \frac{1}{(2\pi)^{\frac{n}{2}}} \int_{\mathbb{R}^{(r,t)}} |g(x)e^{-Iv(u,x)} (e^{-Iv(\xi,x)} - 1)| d^n x \\ &= \frac{1}{(2\pi)^{\frac{n}{2}}} \int_{\mathbb{R}^{(r,t)}} |g(x)e^{-Iv(u,x)}| |e^{-Iv(\xi,x)} - 1| d^n x \\ &\leq \frac{1}{(2\pi)^{\frac{n}{2}}} \int_{\mathbb{R}^{(r,t)}} |g(x)e^{-Iv(u,x)}| (|e^{-Iv(\xi,x)}| + 1) d^n x \\ &\leq \left(\frac{2}{\pi}\right)^{\frac{n}{2}} (2 + |I|^2) \int_{\mathbb{R}^{(r,t)}} |g(x)| d^n x. \end{aligned} \tag{34}$$

Since  $g(x)$  is integrable, then we can use the Lebesgue dominated convergence theorem to get

$$\lim_{\xi \rightarrow 0} |\mathcal{F}_{Cl}\{g\}(u + \xi) - \mathcal{F}_{Cl}\{g\}(u)| = 0, \tag{35}$$

which shows that  $\mathcal{F}_{Cl}\{g\}$  is continuous on  $\mathbb{R}^{(r,t)}$ . □

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